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A Simple Method of Design of Shallow Footings on Expansive Soil

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SUMMARY: A method of design of shallow footings on expansive soil is presented, based on a simplification of actual conditions encountered in practice. Equations are developed to permit the bending moment and stiffness to be determined for either centre or edge heave. A parametive study for centre heave is given, and a comparison is made with the behaviour of a heavily loaded residential raft footing that underwent severe distortion.

1. INTRODUCTION

The routine design of a shallow footing on expansive soil is usually based on the interaction of a loaded footing superimposed on an initially distorted soil surface. Design methods include those of BRAB (1968), Lytton (1977), Walsh (1978), Swinburne (Holland 1981), PTI-Wray (Wray 1980) and Mitchell (1979). The complexity of these methods varies from a simple beam-on-mound analysis to complex finite element computer analysis of a stiffened slab.

Several comparative studies and reviews (e.g. Pidgeon 1980, Holland 1981, Mitchell 1982) have indicated a variation in the answers obtained by each method. The author (Mitchell 1983) has shown that the differences in the design methods appear to be more a result of differences in the input parameters of each method, than the complexity of the method used for the actual interaction analysis. This assumes more importance when it is acknowledged that any method, despite its complexity, could not wholly anticipate the conditions under actual field situations. Complicating factors include those of tree effects, fissures, leaking services, drainage patterns, soil variation etc. Consequently, simplification of actual conditions is necessary for routine design.

One simple design method is presented in this paper. It has been successfully used by the author for the routine design of shallow stiffened footings of structures on expansive soil for at least five years. In particular, it can be conveniently used to examine the effect of the various parameters influencing the problem, and it is shown in this paper to reasonably model the behaviour of a "failed" footing under field conditions.

2. DEVELOPMENT OF A SIMPLIFIED ANALYSIS

The complex analysis of a footing system interacting with a distorted soil surface can be simplified by reducing many of the variables involved, without necessarily resulting in a significant loss of accuracy.

2.1 Geometry

The geometry of the structure can be reduced into rectangular dimensions of length (L) and breadth (B). for complex shapes, the plan of the structure can be divided into overlapping rectangles.

2.2 Load Pattern

A variety of structural loading patterns will occur in practice. However, this complexity can be reduced by simplifying the loads into perimeter line loads (W) and centre line loads (T) and distributed loads (w).

2.3 Shape of Soil Surface

Depending on the conditions occurring at the perimeter of the structure, a variety of shapes of the initially distorted soil surface will occur. In the commonly used design methods, similar convex shapes are adopted for the so-called "centre-heave" or hogging mode of distortion (Fig. 1a). For "edge-heave" or sagging mode however, there is considerable variation among the methods, with some methods adopting a concave shape (fig. 1b) while others adopt an irregular convex shape (Fig. 1c.).

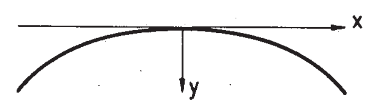

Figure 1a. CENTRE HEAVE

Figure 1b. EDGE HEAVE

Figure 1c. EDGE HEAVE

For a maximim soil swell of Y, the shapes in figures 1a to 1c can be represented by the Lytton equation (Lytton 1977) as follows:-

$$y = \left(\frac{2x}{L}\right)^m Y \quad \text{.........} \quad (1)$$

The exponent m in equation (1) has been found to have a major influence on the footing design (Pidgeon 1980, Holland 1981). The author (Mitchell, 1979,1984) has found from analytical solutions of the steady state Diffusion Equation, that m can be approximated by 1.5L/a, where a is the depth of soil suction change.

2.4 Soil Stiffness

The soil can be modelled by elastic parameters such as Youngs Modulus and Poissons Ratio. The simplest model, however, is the Winkler spring in terms of a constant spring stiffness (k) by equation (2),

$$P = k(y - \delta) \qquad \text{..........(2)}$$

In equation (2), δ is the footing displacement and y is the soil movement.

2.5 Permissible Deflection

The permissible deflection (Δ) of the footing is defined as the maximum footing deflection that will still guarantee a satisfactory level of serviceability of the structure. This has usually been empirically determined from the observed behaviour of structures, so that the allowable deflection ratios (Δ/L) of structures are of the order of 0.0005 for solid brick, 0.0013 for articulated brick, 0.002 for brick veneer, 0.0033 for articulated brick veneer, and 0.005 for timber framed and clad house (e.g. Pidgeon 1980).

2.6 Structural Analysis

The simplest structural analysis is that of a beam-on-mound. Despite this simplification, a beam-on-mound will still model the footing system deformed in cylindrical bending, which is a condition frequently encountered in practice.

For the purposes of determining the soil pressures acting on the footing as it interacts with the soil, let the approximate shape footing (convex for centre heave and concave for edge heave) be defined by an equation similar in type to that representing the soil movement. Letting the exponent of this equation be t, and, as the maximum allowable differential displacement of the footing is Δ, then the footing deformation (S) is:-

$$\delta = \left(\frac{2x}{L}\right)^t \Delta \qquad \text{..........(3)}$$

Equation (3) will not define the exact deflected shape of the footing as it implies the bending moment is zero at x = o for t> 2 However, observations on a distorted raft indicate that equation (3) closely describes the measured pattern of deflections (Cameron 1977) and measurements of deflected floor slabs (Lytton & Meyer 1971) indicate a behaviour approximated by equation (3) with t = 2.

If the footing is in equilibrium, and has lost contact with the soil over a certain length of footing, then three simultaneous equations are established (a) The superstructure loads must equal the soil forces over the length of footing in contact with the soil (b) The bending moment due to the external loads and soil forces must equal to the bending moment corresponding to the curvature of the footing (c) At any point of intersection between the soil profile and the footing profile, the soil displacement equals the footing displacement.

The three equations can be used to determine the distribution of soil forces along the footing length, thereby enabling the determination of the bending moment to permit a footing design.

The two cases, that of a hogging condition (centre heave) and sagging condition (edge heave) will be examined in the following sections.

3. THE CENTRE HEAVE CONDITION

Figure 2 shows a footing interacting with an expansive soil so as to be distorted into centre heave. The soil will be in contact with the footing over a central portion of the footing defined by a parameter C, the support ratio, which is taken as the ratio of the length of the footing in contact with the soil to the total length of the footing.

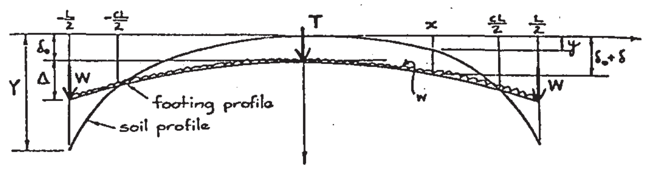

Fig.2: Footing-Soil Interaction for Centre Heave.

The boundary conditions are at x = o, δ = 0 and δ' = o; at x = CL/2, the equations for δ and δ derived for the range 0<x<CL/2 equal δ and δ'' for the range CL/2< x< L/2; and at x = L/2, $\delta = \Delta$.

The three unknowns that must be solved in the analysis are (a) the amount of footing settlement δ_o at x = o (b) the support ratio C and (c) the parameter t defining the deflected shape of the footing. The author (Mitchell 1984) has solved these unknowns by integrating the beam equation for the boundary conditions of the problem. The results are shown in Appendix A by equations A.1 to A.7.

To illustrate the use of the equations in Appendix A, consider a problem with parameters L = 12m, B = 1m, W = 10kN/m, T = O, w = 6.5kPa, Y = 75mm, Δ = 12mm, m = 5 and k = 1000 kPa/m.

Uniform pressure is ω = 8.17kPa.

Try t=1.744, from eq.(A.2) C=0.756.
from eq.(A.5) at x=3 EIδ =323.6 kN-m^3/m
from eq.(A6) EIΔ=1084.2 kN-m^3/m
from eq(A7) at x=3 t=1.744=assumed
Hence t=1.7444, C=0.756
from (A1) δ_o=12.4mm.
Since EIΔ=1084.2 kN-m^3/m

required stiffness EI=90,350 kN-m^2/m.

Equations (A.3) and (A.4) give the bending moment, equation (2) gives the soil reaction, equation (1) gives the soil movement and equation (3) gives the footing displacement, which when added to δ_o gives the total footing movement. The results are shown in Table 1.

TABLE 1: SUMMARY OF RESULTS OF CENTRE HEAVE EXAMPLE

Distance from Centreline (m)	Moment (kN-m/m)	Free Soil Heave (mm)	Footing Movement (mm)	Soil Pressure (kPa)
x = 0	75.6	0	11.2	11.2
x = 1.2	72.2	0	11.9	11.9
x = 2.4	60.9	0.8	13.9	13.1
x = 3.6	40.8	5.8	16.1	10.3
x = 4.8	16.7	24.6	19.3	0
x = 6.0	0	75.0	23.2	0

4.0 THE EDGE HEAVE CONDITION

Figure 3 shows a footing interacting with an expansive soil so as to be distorted into edge heave. Attention is restricted in this paper to the distorted soil surface being a concave shape, although a shape shown in figure 1c. could also be considered. When the soil is in a concave shape, the soil will be in contact with the footing over the portion at the edges of the footing defined by the support ratio (C). The boundary conditions are at x=0, $\delta = 0$, and $\delta' = 0$; at $x = (1-C)L/2$ the equations for δ and δ', derived for $0 < x < (1-C)L/2$ equal δ and δ' for $(1-C)L/2 < x < L/2$; and at $x = L/2$, $\delta = \Delta$

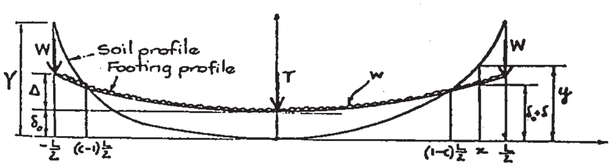

Fig. 3: Footing-Soil Interaction for Edge Heave.

The three unknowns are (a) the amount of lift-off, (δ_o) between the footing and the soil at x=0 (b) the support ratio C, and (c), the parameter t defining the deflected shape of the footing. In a similar manner to that adopted for centre heave, the author (Mitchell 1984) has solved for these unknowns by integrating the beam equation for the boundary conditions of the problem. The results are shown in Appendix B by equations B.1 to B.8.

To illustrate the analysis, consider a problem with L = 12m, B=1m, W = 10 kN/m, T = 0, m = 6.5 kPa, Y = 75mm, Δ= 12mm, m = 5 and k = 1000 kPa/m.

The uniform pressure is $\omega = 8.17$ kPa.

Try t=1.722, from eq.(B.21)	C=0.393
from eq(B.5) at x=3	$EI\delta = -367.8$ kN-m^3/m
from eq.(B.7),	$EI\Delta = -1214.2$ kN-m^3/m
from eq.(B.8) at x=3	t=1.722 = assumed
Hence t=1.722	C=0.393
from eq.(B.1)	$\delta_o = 1.1$mm.

Since $EI\Delta = 1214.2$ kN-m^3/m,
Required stiffness EI=101,183 kN-m^2/m

Equations (B.4) and (B.6) give the bending moment, equation (2) gives the soil reaction, equation (1) gives the soil movement, and equation (3) the footing displacement, which, when added to δ_o gives the total footing movement. Table 2 gives a summary of the results.

TABLE 2: SUMMARY OF RESULTS FOR EDGE HEAVE EXAMPLE

Distance from Centre-line (m)	Moment (kN-m/m)	Free Soil Heave (m)	Footing Move-Ment (mm)	Soil Pressure (kPa)
x = 0	-86.5	0	1.1	0
x = 1.2	-81.8	0	1.9	0
x = 2.4	-67.8	0.8	3.6	0
x = 3.6	-44.4	5.8	6.1	0
x = 4.8	-14.0	24.6	9.3	15.3
x = 6.0	0	75.0	13.1	61.9

5. PARAMETRIC STUDY FOR CENTRE HEAVE

Figures 4 to 11 show the solution for the centre heave bending moment in a footing in which one of the parameters L, W, w, T, m, k, Y and Δ is varied with the others remaining constant. The bending moment is plotted as a function of distance from the centre of the footing, with the full bending moment curve being symmetrical about the centre line.

Figure 4 shows that as the free soil swell (Y) increases, the bending moment increases. The rate of increase of bending moment with soil swell is large for small soil swells, the rate decreasing at large soil swells. Figure 5 shows that a large variation in swelling stiffness (k) does not lead to a significant variation of maximum bending moment. Figure 6 shows that an increase in the shape factor (m) of the soil leads to a reduction in bending moment. Fig.7 shows that as Δ increases, moment decreases. Figures 8 and 9 show that the bending moment is very sensitive to the magnitude of perimeter load (W), and uniform pressure (w). The effect of an increasing load (T) placed at the centre of the footing is to decrease the bending moment as shown in figure 10.

The effect of the footing length (L) on the bending moment is shown in figure 11. At small footing lengths, an increase in length causes an increase in bending moment, with the maximum moment occurring at the footing centre. However, for longer footing lengths, a peak is reached beyond which a slight fall in the magnitude of maximum moment occurs with an increase in footing length. The position of the maximum bending moment departs from the footing centre and for very long footings, the maximum bending moment occurs only a few metres from the footing edge.

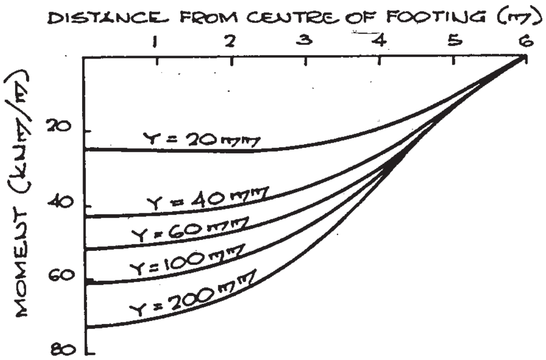

Fig.4: Effect of Soil Swell for L=12m, W=10kN/m, w=6.5kPa, m=8, k=1000kPa/m, Δ = 12mm, T=o.

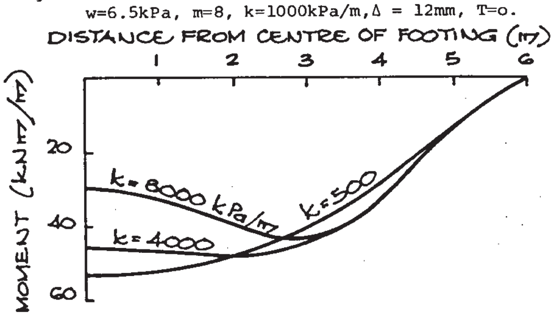

Fig.5: Effect of Swell Stiffness for L=12m, W=10kN/m, w=6.5kPa, m=8, Y=75mm, Δ=12mm, T=o.

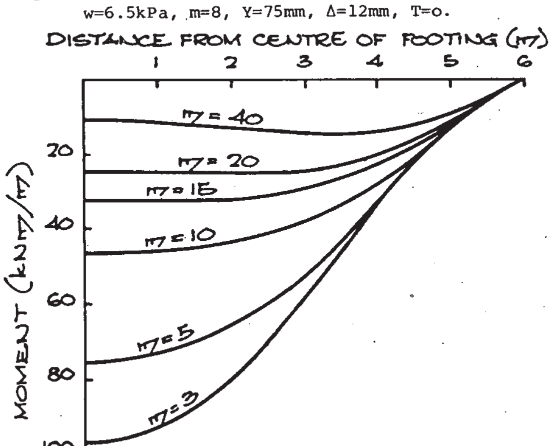

Fig.6: Effect of Shape Factor of L=12m, W=10kN/m, w = 6.5 kPa, k=1000kPa/m, Y=75mm, Δ=12mm, T=o.

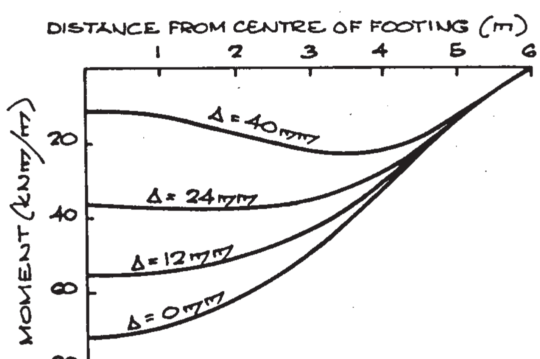

Fig.7: Effect of Permissible Defection for L=12m, W=10kN/m, w=6.5kPa, m=8, k=1000kPa/m, Y=75mm, T=0.

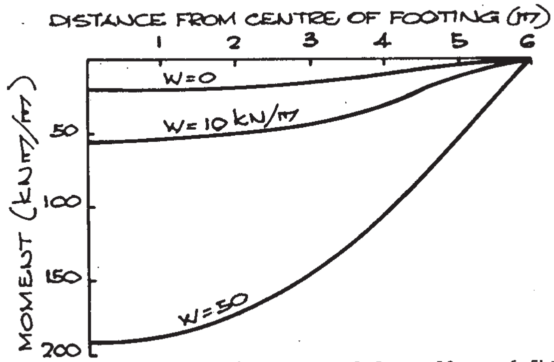

Fig.8: Effect of Perimenter Load for L=12m, w=6.5kPa, m=8, k=1000kPa/m, Y=75, Δ =12mm, T=o.

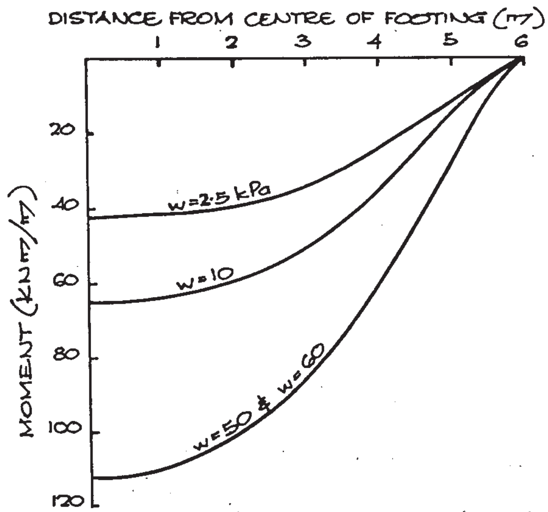

Fig.9: Effect of Uniform Load for L=12m, W=10kN/m, m=8, k = 1000kPa/m, Y = 75mm, Δ =12mm, T=o.

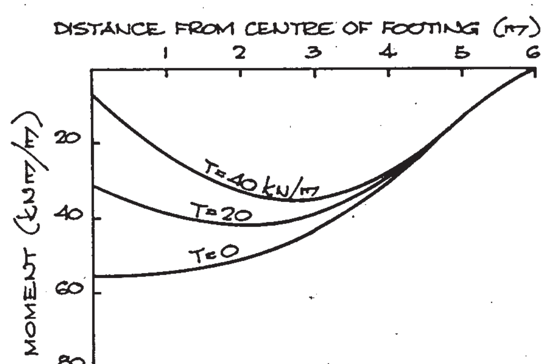

Fig.10: Effect of Central Load for L=12m, W=10kN/m, w=6.5kPa, m=8, k=1000kPa/m, Y=75mm, Δ = 12mm. T =0.

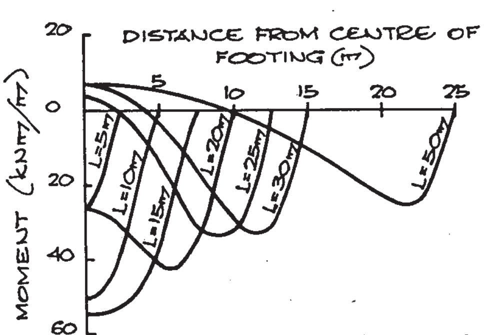

Fig.11: Bending Moments for Various Lengths. W=10kN/m, w=6.5kPa, m=0.67L, k= 1000kPa/m, Y=75mm, Δ = 0.001L, T=o.

CASE EXAMPLE

To illustrate the appropriateness of the method of analysis in modelling the observed behaviour of footing systems of structures under conditions encountered in practice, a comparison is made with the observed distortion of a raft footing of a house constructed in Newton, South Australia. The house was two storey solid brick (estimated loading W = 30 kN/m, T = 30 kN/m, w = 3 kPa) of "L-shaped" configuration, and the raft was constructed on a wet soil in June 1978.

The sub-beams of the raft were 400mm wide x 500mm deep, reinforced with 4 No. 12 mm dia. bars of 410 MPa steel top and bottom, cast integrally with a 100 mm floor slab reinforced with F62 (6mm wire of 450 MPa steel at 200mm centres both ways). The ultimate moment capacity of the raft can be calculated as 55 kNm/m.

During March 1979, cracking had developed in the structure, this being concentrated at the end of a 14.05m x 5.34m rectangular section. Here, shrinkage of the soil occurred during the dry season subsequent to construction. On this rectangular section, two sub-beams were constructed in the long direction, and the mode of bending was cylindrical hogging in the long direction. A level traverse indicated a raft deflection of 15 mm over a distance of 2m, and the raft slab exhibited cracking consistent with a rupture of the raft. A plastic hinge had developed 1.7m to 3m from the edge of the raft, and this was associated with a noticeably abrupt change in curvature of the raft.

The measured deflection of 15 mm was approximately double that taken to be allowable for solid brick construction, since, for a permissible deflection of $\Delta = 0.0005L$, only 7 mm can be tolerated over the 14m length. The results of the level traverse and observed raft cracks are shown in figure 12.

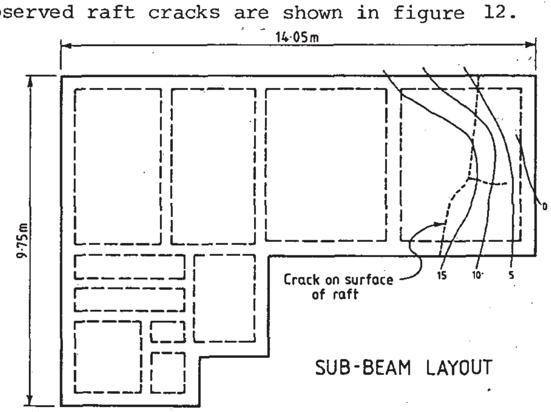

Figure 12. Observed distortion of raft footing of Newton House.

Because the raft developed a plastic hinge, the raft must have been subjected to a bending moment, at least at the location of the hinge, in excess of the ultimate bending moment capacity of the raft. Because a hinge developed, the raft followed the distorted shape of the ground surface, so that a soil movement of Y = 20mm must be appropriate. The soil profile contained weathered sandstone below 1.2m, so that the soil suction changes and resulting soil movements must have been contained in the upper a = 1m of the profile. This implies that the shape factor m is from $m = 1.5L/a = (1.5 \times 14.05)/1.0 = 21$. A swelling stiffness of k = 2500 kPa/m can be taken, as it is a value commonly used in the Adelaide area for a house loading of this type. Also, on the basis of Figure 5, the value of swelling stiffness does not appear to be a sensitive parameter.

Hence the parameters defining the problem are L = 14.05m, B = 5.34m, W = 30 kN/m, T = 30 kN/m, w = 3 kPa, Y = 20mm, Δ= 7mm, m = 21 and k = 2500 kPa/m. Equations A.1 to A.7 are used to calculate the bending moment in the footing. The values of C and t are found to be C = 0.98, t = 2.32. The predicted bending moment distribution is shown in figure 13.

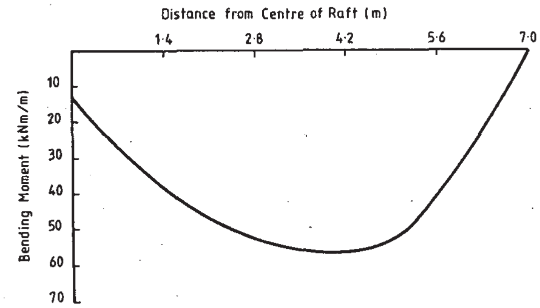

Figure 13. Predicted bending moment of raft footing of Newton house.

It is seen that the predicted maximum bending moment occurs at a distance of 2.8m from the end of the footing, which corresponds to the approximate location of the plastic hinge that occurred at the site. The maximum bending moment was found to be 56 kNm/m which was slightly in excess of the ultimate bending moment capacity of the footing, confirming the development of the hinge.

The analysis is thus shown to describe the observed behaviour of the footing reasonably well.

CONCLUSION

A method of analysis is presented for the design of a shallow footing on expansive soil. The method is based on a simplification of the structure geometry, load and permissible deflection, and is derived from an integration of the beam-on-Winkler mound equation.

Algebraic expressions for the bending moment at any point on the footing, together with the required stiffness have been obtained for both centre and edge heave. A parametric study has revealed that each of the design parameters (i.e. structure geometry, loads, soil movement, permissible deflection, shape of initial soil surface and swelling stiffness) have a variable effect on the bending moment in the footing.

The behaviour of a heavily loaded residential raft footing undergoing severe distortion is shown to be successfully modelled by the analysis. This indicates that although the method involves a simplification of the actual field conditions, a significant loss of accuracy does not necessarily occur.

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APPENDIX A

Equations for Centre Heave ($C < 1$). Refer to Fig. 2., with ω is the uniform pressure.

$$\delta_o = C^m Y - C^t \Delta \qquad \text{.......... A1}$$

C is the solution of

$$\frac{\omega}{kY} = C^{m+1}\left(\frac{m}{m+1}\right) - C^{t+1}\left(\frac{\Delta}{Y}\right)\left(\frac{t}{t+1}\right) \qquad \text{.......... A2}$$

For $CL/2 < x < L/2$

$$M_x = WB\left(\frac{L}{2} - x\right) + \frac{wB}{2}\left(\frac{L}{2} - x\right)^2 \qquad \text{.......... A3}$$

For $o < x < CL/2$

$$\begin{aligned} M^- = M_x - kB\Bigg[& \delta_o \frac{(CL)^2}{8} + \left(\frac{2}{L}\right)^t \left(\frac{CL}{2}\right)^{t+2} \frac{\Delta}{t+2} - \left(\frac{2}{L}\right)^m \left(\frac{CL}{2}\right)^{m+2} \frac{Y}{m+2} \\ & - \delta_o \left(\frac{CL}{2}\right) x - \left(\frac{2}{L}\right)^t \left(\frac{CL}{2}\right)^{t+1} \frac{\Delta x}{t+1} + \left(\frac{2}{L}\right)^m \left(\frac{CL}{2}\right)^{m+1} \frac{Yx}{m+1} \\ & + \delta_o \frac{x^2}{2} + \left(\frac{2}{L}\right)^t \frac{x^{t+2}\Delta}{(t+1)(t+2)} - \left(\frac{2}{L}\right)^m \frac{x^{m+2} Y}{(m+1)(m+2)} \Bigg] \end{aligned} \qquad \text{.......... A4}$$

$$\begin{aligned} EI\delta = {} & WB\frac{x^2}{2}\left(\frac{L}{2} - \frac{x}{3}\right) + wB\frac{x^2}{4}\left(\frac{L^2}{4} - L\frac{x}{3} + \frac{x^2}{6}\right) \\ & - kB\Bigg[\frac{\delta_o}{4}\left(\frac{CL}{2}\right)^2 x^2 + \left(\frac{2}{L}\right)^t \left(\frac{CL}{2}\right)^{t+2} \frac{\Delta x^2}{2(t+2)} - \left(\frac{2}{L}\right)^m \left(\frac{CL}{2}\right)^{m+2} \frac{Yx^2}{2(m+2)} \\ & - \delta_o \left(\frac{CL}{2}\right)\frac{x^3}{6} - \left(\frac{2}{L}\right)^t \left(\frac{CL}{2}\right)^{t+1} \frac{\Delta x^3}{6(t+1)} + \left(\frac{2}{L}\right)^m \left(\frac{CL}{2}\right)^{m+1} \frac{Yx^3}{6(m+1)} \\ & + \delta_o \frac{x^4}{24} + \left(\frac{2}{L}\right)^t \frac{x^{t+4}\Delta}{(t+1)(t+2)(t+3)(t+4)} - \left(\frac{2}{L}\right)^m \frac{x^{m+4} Y}{(m+1)(m+2)(m+3)(m+4)} \Bigg] \end{aligned} \qquad \text{.......... A5}$$

$$\begin{aligned} EI\Delta = {} & WB\frac{L^3}{24} + wB\frac{L^4}{128} - kB\Bigg[\frac{\delta_o}{6}\left(\frac{CL}{2}\right)^3 \left(\frac{L}{2}\right) \\ & + \left(\frac{2}{L}\right)^t \left(\frac{CL}{2}\right)^{t+3} \frac{\Delta}{2(t+3)} \left(\frac{L}{2}\right) - \left(\frac{2}{L}\right)^m \left(\frac{CL}{2}\right)^{m+3} \frac{Y}{2(m+3)} \left(\frac{L}{2}\right) - \frac{\delta_o}{24}\left(\frac{CL}{2}\right)^4 \\ & - \left(\frac{2}{L}\right)^t \left(\frac{CL}{2}\right)^{t+4} \frac{\Delta}{6(t+4)} + \left(\frac{2}{L}\right)^m \left(\frac{CL}{2}\right)^{m+4} \frac{Y}{6(m+4)} \Bigg] \end{aligned} \qquad \text{.......... A6}$$

t at a selected value of x is the solution of

$$\frac{EI\delta}{EI\Delta} = \left(\frac{2x}{L}\right)^t \qquad \text{.......... A7}$$

APPENDIX B

Equations for Edge Heave. Refer to Fig. 4, with ω is the uniform pressure.

$$\delta_o = (1-C)^m Y - (1-C)^t \Delta \qquad \text{.......... B1}$$

For δo >o C is the solution of

$$\frac{\omega}{kY} = \left[1-(1-C)^m(1+mC)\right]\frac{1}{m+1} + \left[(1-C)^t(1+tC)-1\right]\left(\frac{\Delta}{Y}\right)\frac{1}{t+1} \qquad \text{.......... B2}$$

For $\delta_o = 0$,

$$C = 1-\left(\frac{\Delta}{Y}\right)^{\frac{1}{m-t}} \qquad \text{.......... B3}$$

For o <x < (1-C) L/2

$$\begin{aligned} M^+ = {} & WB\left(\frac{L}{2}-x\right) + \frac{wB}{2}\left(\frac{L}{2}-x\right)^2 \\ & - kB\left[\frac{YL^2}{4(m+2)}(1-(1-C)^{m+2}) - \frac{\Delta L^2}{4(t+2)}(1-(1-C)^{t+2})\right. \\ & - \frac{\delta_o L^2}{8}(1-(1-C)^2) - \frac{YxL}{2(m+1)}(1-(1-C)^{m+1}) \\ & \left. + \frac{\Delta xL}{2(t+1)}(1-(1-C)^{t+1}) + \frac{\delta_o LxC}{2}\right] \end{aligned} \qquad \text{.......... B4}$$

$$\begin{aligned} EI\delta = {} & \frac{WBL}{4}x^2 - \frac{WB}{6}x^3 + \frac{wBL^2}{16}x^2 - \frac{wBL}{12}x^3 + \frac{wB}{24}x^4 \\ & - kB\left[\frac{YL^2x^2}{8(m+2)}(1-(1-C)^{m+2}) - \frac{\Delta L^2x^2}{8(t+2)}(1-(1-C)^{t+2})\right. \\ & - \frac{\delta_o L^2x^2}{16}(1-(1-C)^2) - \frac{YLx^3}{12(m+1)}(1-(1-C)^{m+1}) \\ & \left. + \frac{\Delta Lx^3}{12(t+1)}(1-(1-C)^{t+1}) + \frac{\delta_o LCx^3}{12}\right] \end{aligned} \qquad \text{.......... B5}$$

and for $\frac{(1-C)L}{2} < x < \frac{L}{2}$,

$$\begin{aligned} M^+ = {} & WB\left(\frac{L}{2}-x\right) + \frac{wB}{2}\left(\frac{L}{2}-x\right)^2 \\ & - kB\left[\frac{L^2Y}{4(m+2)} - \frac{L^2\Delta}{4(t+2)} - \frac{\delta_o L^2}{8}\right. \\ & - \frac{LxY}{2(m+1)} + \frac{Lx\Delta}{2(t+1)} + \frac{\delta_o xL}{2} \\ & \left. + \left(\frac{2}{L}\right)^m \frac{x^{m+2}Y}{(m+1)(m+2)} - \left(\frac{2}{L}\right)^t \frac{x^{t+2}\Delta}{(t+1)(t+2)} - \frac{\delta_o x^2}{2}\right] \end{aligned} \qquad \text{.......... B6}$$

$$\begin{aligned} EI\Delta = {} & \frac{WBL^3}{24} + \frac{wBL^4}{128} \\ & - kB\left[\frac{(2m+9)YL^4}{96(m+3)(m+4)} - \frac{(2t+9)\Delta L^4}{96(t+3)(t+4)} - \frac{\delta_o L^4}{128}\right. \\ & - \frac{(1-C)^{m+3}YL^4}{32(m+3)} + \frac{(1-C)^{t+3}\Delta L^4}{32(t+3)} + \frac{(1-C)^3\delta_o L^4}{96} \\ & \left. + \frac{(1-C)^{m+4}YL^4}{96(m+4)} - \frac{(1-C)^{t+4}\Delta L^4}{96(t+4)} - \frac{(1-C)^4\delta_o L^4}{384}\right] \end{aligned} \qquad \text{.......... B7}$$

t at a selected value of x is the solution of

$$\frac{EI\delta}{EI\Delta} = \left(\frac{2x}{L}\right)^t \qquad \text{.......... B8}$$